Lecture 5: An Introduction to R

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Contents

1 Getting Started with R

1.1 What is R?

R is a free software environment and programming language for statistical computing and graphics. The website for the R project is `https://www.r-project.org/`.

A brief history of development of R

- It originated from Bell Laboratories in the 1970s, as the S language, from which the commercial version S-Plus was developed in 1987.
- R was initially developed by Robert Gentleman and Ross Ihaka at the University of Auckland, New Zealand in 1996.
- Since its first release in 2000, the development of the R project has been tremendous in the last two decades.

Why do we choose R other than other econometric software?

- It is open source, free to download.

- It has a huge number of packages that can implement almost all state-of-art statistical techniques.
- It has a powerful and flexible capabilities of making graphs.
- It is a programming language designed specifically for statistics, enabling you to accomplish almost anything a programming language can do for statistics.

1.2 Installation

Install R

The installation files can be downloaded from `https://mirrors.tuna.tsinghua.edu.cn/CRAN/`. You can download the installation files for Windows, OS X, and Linux(ubuntu).

Install RStudio

The base R comes with a simple Graphic User Interface (GUI). RStudio supplies with a more user-friendly GUI and provides other powerful functionalities, such as writing dynamic documents with nitro and rmarkdown.

- RStudio can be downloaded from `https://www.rstudio.com/products/rstudio/download/`
- The window of RStudio looks like Figure 1

1.3 Packages

The R installation files install the core packages that support very basic functions. One of the strength of R is that there are many contributed packages written by the huge community of R users.

To install a contributed package, we use the command `install.packages("names of packages")`. After installing a package, we need to invoke it every time we use it by the command `library(name of a package)`. In this course, for example, we need to install a package called `AER` (Applied Econometrics with R).

Type the following code in the "Console" window in RStudio.

```
# Install packages
install.packages("AER")
```

Upon typing this command, a window jumps up for you to choose a mirror. From the list, choose China[Beijing]. R will automatically download and install this package from the server. Very likely, when this is the first package you install in R, R will also download other packages on which installing the `AER` package depends. In the console, you should see the following messages.

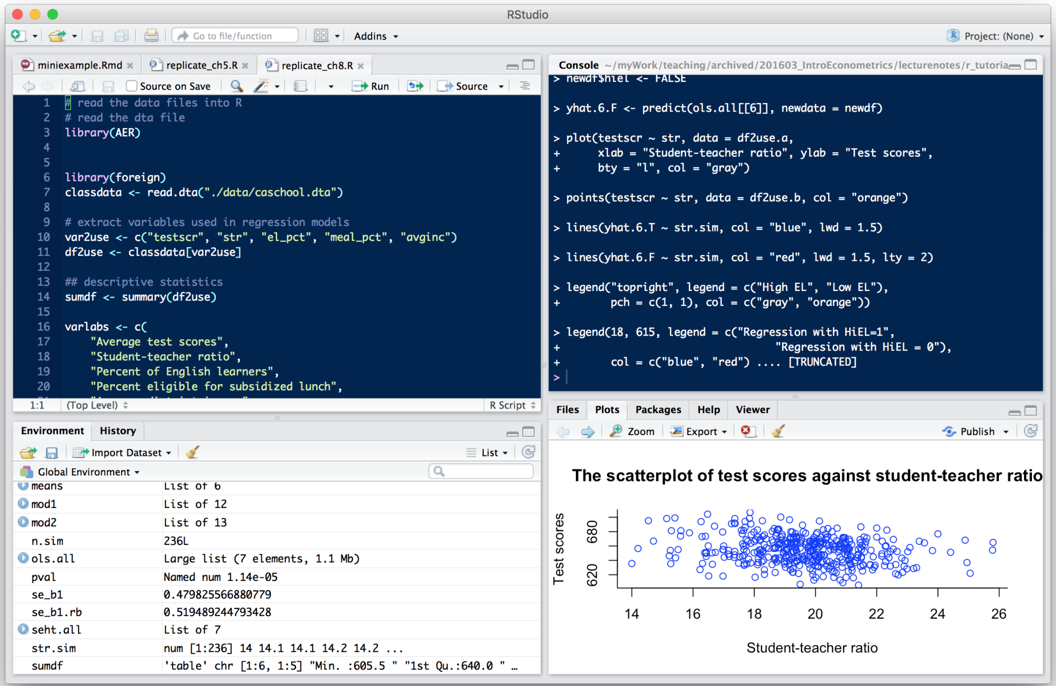


Figure 1: The Window of RStudio

```
trying URL 'https://mirrors.tuna.tsinghua.edu.cn/CRAN/bin/macosx/mavericks/contrib/3.3/AER_1.
Content type 'application/octet-stream' length 2442603 bytes (2.3 MB)
==================================================
downloaded 2.3 MB


The downloaded binary packages are in
/var/folders/rd/53x_sgqd3yj6wghsyyy4n0vh0000gn/T//RtmpF3tVDW/downloaded_packages
```

In RStudio, you can install packages from the "Tools" menu and click "Install Packages".

When we need to use the AER package, type `library(AER)` in the console. And we can check whether this package is loaded using `search()`.

```
# Load packages
library(AER)

# Check packages loaded
search()
```

```
 [1] ".GlobalEnv"        "package:foreign"   "package:AER"
 [4] "package:survival"  "package:sandwich"  "package:lmtest"
 [7] "package:zoo"       "package:car"       "ESSR"
[10] "package:stats"     "package:graphics"  "package:grDevices"
```

```
[13] "package:utils"     "package:datasets"  "package:methods"
[16] "Autoloads"         "package:base"
```

It shows that besides the AER package, there are other packages in the "global" environment, which are the core packages loaded automatically when opening R.

1.4 Help

- R has easy helping facilities. The help information of any function can be found by type either `help()` or `?`.
- If you cannot remember the accurate name of a function, you can even guess by using `help.search()` or `??` or `apropos()`.
- Any time you encounter a problem using R which cannot be solved by `help` command, there are at least two places you can resort to.
  - The mailing list of R: http://www.r-project.org/mail.html
  - Google or bing: quite often you will get an answer to your question in the website of http://stackoverflow.com/.

2 Basics

2.1 R as a calculator

Standard arithmetic operators

R supports the following arithmetic operators

`+, -, *, /, ^, %%, %/%`

Hence,

```
## R as a calculator ------------------------------

#+ Binary operations
1 + 2; 2*3; 2^3; 5/2;
5 %% 2  # get x mod y
5 %/% 2 # get the integer division
```

```
[1] 3
[1] 6
[1] 8
[1] 2.5
[1] 1
```

```
[1] 2
```

Mathematical functions

R also have many built-in mathematical functions, such as, `log()`, `exp()`, `sin()`, `sqrt()`, `min()`, etc.

```
# Use built-in functions
log(exp(sin(pi/2)^2) * exp(cos(pi/3)^2))
```

```
[1] 1.25
```

2.2 Vector operations

Vector is the basic unit in R, from which other data structures, for example, `matrix`, `factor`, `list`, `data.frame`, are built upon.

Generate a vector

A vector can be generated by the function `c()`, which can also be used to concatenate two vectors

```
## Vector operations ------------------------------

# Create a vector with c()
x <- c(0.3, 1.5, 7.3, 2)
y <- c(3, 2, 1)
z <- c(x, y)
z
```

```
[1] 0.3 1.5 7.3 2.0 3.0 2.0 1.0
```

The symbol `<-` is to assign a value to a variable. You can also use `=` to assign values, but `<-` is more commonly used by convention and `=` is used within a function calling for assigning values to the arguments of the function.

Note that by concatenating `x` and `y`, integers are converted to floating point numbers. That means the elements in a vector must have the same mode (data types), including `numeric`, `character`, and `logical`.

```
# Vectors with different data types
student.names <- c("John", "Mary", "Bob", "Ann")
student.male <- c(TRUE, FALSE, TRUE, FALSE)
student.age <- c(20, 19, 21, 20)
```

```
class(student.names)
class(student.male)
class(student.age)

students <- c(student.names, student.male, student.age)
students
```

```
[1] "character"
[1] "logical"
[1] "numeric"
 [1] "John"  "Mary"  "Bob"   "Ann"   "TRUE"  "FALSE" "TRUE"  "FALSE" "20"
[10] "19"    "21"    "20"
```

Patterned vectors

A vector can also be generated by the functions, like `rep()`, `seq()`, and `:`.

- `seq()` generates a vector by some patterns and `a:b` is a shorthand for `seq(from=a, to=b, by=1)`.

  ```
  # Create a sequence
  even <- seq(from = 2, to = 20, by = 2)
  even
  years <- 1995:2005
  years
  ```

  ```
  [1]  2  4  6  8 10 12 14 16 18 20
  [1] 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005
  ```

- `rep()` generates a vector by repeating some values

  ```
  # Create repetition
  ones <- rep(1, times = 10)
  ones
  rep13 <- rep(1:3, times = 3, each = 2)
  rep13
  ```

  ```
  [1] 1 1 1 1 1 1 1 1 1 1
  [1] 1 1 2 2 3 3 1 1 2 2 3 3 1 1 2 2 3 3
  ```

Vector operations

Arithmetic operators and mathematical functions can be applied to vector in an element-by-element way in R.

Let's first draw random numbers for the uniform distribution $x \sim Uniform(0, 1)$. The length of x is 10. We can use the `length()` function to check the length of a vector.

```
# Draw a random vector
x <- runif(10); x
length(x)
```

```
 [1] 0.865327406 0.384586069 0.033214976 0.377298041 0.991853392 0.191534136
 [7] 0.281390713 0.003944363 0.615192923 0.326255273
[1] 10
```

The arithmetic operations and built-in math functions are all applied for each element of a vector.

```
2 * x + 3
log(x)
```

```
[1] 4.730655 3.769172 3.066430 3.754596 4.983707 3.383068 3.562781 3.007889
[9] 4.230386 3.652511
[1] -0.144647339 -0.955587668 -3.404754418 -0.974719845 -0.008179973
[6] -1.652689232 -1.268011138 -5.535467872 -0.485819365 -1.120075159
```

If two vectors with different lengths are computed within one operation, the elements of the vector with a shorter length will be used in an iterated way. We must keep in mind this feature of R, which in some cases may give rise to unintended results.

```
y <- runif(5)
x + y
```

```
[1] 1.7021802 0.9223989 0.4507015 1.3400343 1.3665239 1.0283869 0.8192036
[8] 0.4214308 1.5779292 0.7009257
```

Selecting elements in a vector

Element(s) in a vector can be selected by `[position]`, in which `position` can be a vector indicating the position of each element in a vector, a negative value to exclude an element with the corresponding position, and a condition to select elements satisfying the condition.

```
# Selecting elements in a vector
x[1:5]
x[c(1, length(x))]
x[-4]
x[x > 0.5]
```

```
[1] 0.86532741 0.38458607 0.03321498 0.37729804 0.99185339
[1] 0.8653274 0.3262553
[1] 0.865327406 0.384586069 0.033214976 0.991853392 0.191534136 0.281390713
```

```
[7] 0.003944363 0.615192923 0.326255273
[1] 0.8653274 0.9918534 0.6151929
```

Instead of selecting elements in a vector by their positions, we can also give each element a particular name so that we can use their names to choose elements.

```
student.names
student.age
# Give elements names
names(student.age) <- student.names
student.age
student.age[c("John", "Bob")]
```

```
[1] "John" "Mary" "Bob"  "Ann"
[1] 20 19 21 20
John Mary  Bob  Ann
  20   19   21   20
John  Bob
  20   21
```

2.3 Matrices

Create a matrix

We can create a matrix with the `matrix()` function, in which the first argument is a vector. We specify the two dimensions by the arguments of `nrow` and `ncol`. By default, `matrix()` arranges all the elements of the vector in its first argument into a matrix by column. We can change it by adding `byrow=TRUE`.

```
# Create a matrix
A <- matrix(1:12, nrow = 3, ncol = 4); A
matrix(1:12, nrow = 3, ncol = 4, byrow = TRUE)
```

```
     [,1] [,2] [,3] [,4]
[1,]    1    4    7   10
[2,]    2    5    8   11
[3,]    3    6    9   12
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    4
[2,]    5    6    7    8
[3,]    9   10   11   12
```

We can also juxtapose vectors of the same length to create a matrix by `cbind()`, or stack over vectors by `rbind()`.

```
# Create a matrix by combining vectors
```

```
a <- 1:4; b <- 2:5; c <- 3:6
cbind(a, b, c)
rbind(a, b, c)
```

```
     a b c
[1,] 1 2 3
[2,] 2 3 4
[3,] 3 4 5
[4,] 4 5 6
  [,1] [,2] [,3] [,4]
a    1    2    3    4
b    2    3    4    5
c    3    4    5    6
```

Like vectors, we can also give each row and each column in a matrix their specific names. Here we use the function of `paste()` to combine two (character) vectors together to generate a new character vector.

```
# Give names to rows and columns
rownames(A) <- paste("X", 1:3, sep = "")
colnames(A) <- paste("Y", 1:4, sep = "")
A
```

```
   Y1 Y2 Y3 Y4
X1  1  4  7 10
X2  2  5  8 11
X3  3  6  9 12
```

Select elements

We select elements from a matrix using `[rows, cols]`. `rows` and `cols` are two vectors to set the rows and columns of elements to be selected.

```
# Selecting elements in a matrix
A[1, 3]
A["X1", "Y3"]
A[1:3, c(2, 4)]
A[, 2]
A[3, ]
```

```
[1] 7
[1] 7
   Y2 Y4
X1  4 10
X2  5 11
```

```
X3   6 12
X1 X2 X3
 4  5  6
Y1 Y2 Y3 Y4
 3  6  9 12
```

Matrix operations

We can do all matrix operations that we have reviewed in Lecture 4.

- Transpose

  ```
  t(A)
  ```

  ```
     X1 X2 X3
  Y1  1  2  3
  Y2  4  5  6
  Y3  7  8  9
  Y4 10 11 12
  ```

- Matrix multiplication

  There are two types of matrix multiplication. The `*` operator computes the element-by-element multiplication (Hadamard product), while the operator `%*%` computes matrix multiplication in the form of inner products of row and column vectors.

  When we do either type of matrix multiplication, we should always check whether the two matrices are conformable to do so. If not, R will give you an error message. We can use the function `dim()` to see the dimensions of a matrix.

  ```
  B <- matrix(1:8, nrow = 4)
  A * B # element-by-element multiplication
  dim(A)
  dim(B)
  ```

  ```
  Error in A * B : non-conformable arrays
  [1] 3 4
  [1] 4 2
  ```

  ```
  A %*% B
  ```

  ```
     [,1] [,2]
  X1   70  158
  X2   80  184
  X3   90  210
  ```

- Inverse matrix

We use the function solve(A) to get the inverse matrix of $\mathbf{A}$.

```
A <- matrix(rnorm(9), nrow = 3)
B <- solve(A)
A %*% B
```

```
              [,1] [,2] [,3]
[1,]  1.000000e+00    0    0
[2,] -1.110223e-16    1    0
[3,]  9.020562e-17    0    1
```

Notice that the resultant matrix is not exactly an identity matrix, in which some off-diagonal elements are very small non-zero numbers. These are the rounding errors stemming from conversion between binary bits (a sequence of 0 and 1) to floating point numbers.

solve() can also be used to solve a system of linear equations, such as,

$$
\begin{aligned}
3x + 2y - z &= 1 \\
2x - 2y + 4z &= -2 \\
-x + \frac{1}{2}y - z &= 0
\end{aligned}
$$

to which the solution is $x = 1, y = -2, z = -2$.

The system of equations can be written in matrix notation as

$$
\begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 4 \\ -1 & \frac{1}{2} & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}
$$

```
A <- cbind(c(3, 2, -1),
   c(2, -2, 0.5),
   c(-1, 4, -1))
B <- c(1, -2, 0)
solve(A, B)
```

```
[1]  1 -2 -2
```

Diagonal matrix

The function diag() can create a diagonal matrix.

```
diag(1:3)
```

```
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    2    0
```

```
[3,]    0    0    3
```

An identity matrix is a special case of a diagonal matrix.

```
diag(3)
```

```
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

Higher-dimensional array

Vectors and matrices are special cases of arrays. The former is one-dimensional array, and the latter is two-dimensional. We can also create higher-dimensional arrays by `array()`.

```
array(1:18, dim = c(3, 3, 2))
```

```
, , 1

     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9

, , 2

     [,1] [,2] [,3]
[1,]   10   13   16
[2,]   11   14   17
[3,]   12   15   18
```

2.4 List

Vectors, matrices, and arrays are all the ways of R to store data. However, their limitation is obvious, all elements in a vector or a matrix must be of the same type. To overcome this limitation, R uses another way to store data, called a `list`.

Here is how we create a list, which consists of three components, a character vector `chr`, a numeric vector `num`, and a logical vector `boo`. Note that the lengths of all components do not need to be equal.

```
mylist <- list(chr = c("a", "b", "c", "d"),
       num = 1:10,
       boo = c(TRUE, FALSE, FALSE, TRUE))
```

```
mylist
```

```
$chr
[1] "a" "b" "c" "d"

$num
 [1]  1  2  3  4  5  6  7  8  9 10

$boo
[1]  TRUE FALSE FALSE  TRUE
```

To select a component, we use the `$=operator or =[[]]`.

```
mylist$chr
mylist[[2]][3:6]
mylist[["boo"]][-1]
```

```
[1] "a" "b" "c" "d"
[1] 3 4 5 6
[1] FALSE FALSE  TRUE
```

3 Data Management in R

R use data frames as its main device to save a whole data set, especially data read from an external file. A data frame is a mixture of a list and a matrix. As a list, a data frame can include different types of data and use the `$` or `[[]]` operator to select a component that is a variable in the data set. As a matrix, all variables in a data frame should have the same length and are arranged in a matrix format.

3.1 Create a data frame

We can manually create a data frame object, convert a matrix to a data frame object, or read data in an external file into R and save them in a data frame object.

Create a data frame manually

```
mydata <- data.frame(X = 1:5, Y = letters[1:5], Z = rep(c(TRUE, FALSE), length = 5)); mydata
```

```
  X Y     Z
1 1 a  TRUE
2 2 b FALSE
3 3 c  TRUE
4 4 d FALSE
```

```
5 5 e  TRUE
```

Convert a matrix to a data frame

We use `as.data.frame()` to convert a matrix to a data frame. In creating the matrix, we use `sample.int()` that is a special case of the function `sample()` to draw random samples from a vector.

```
A <- matrix(sample.int(100, size = 20), nrow = 5)
A.df <- as.data.frame(A); A.df
```

```
  V1 V2 V3 V4
1  3 30 91 79
2 84 58 24 27
3 19 77 98 70
4 94 46 13 38
5 35 85 11 92
```

We can assign each variable (column) a name. Here we use the function `paste()` to combine a string `VAR` with each element of the vector `1:4`, joined with `_`.

```
names(A.df) <- paste("VAR", 1:4, sep = "_"); A.df
```

```
  VAR_1 VAR_2 VAR_3 VAR_4
1     3    30    91    79
2    84    58    24    27
3    19    77    98    70
4    94    46    13    38
5    35    85    11    92
```

3.2 Read data from a file

Suppose we have a data file, `mydata.txt`. We can read the data directly from the file using the function `read.table()`. Upon reading the data into R, we should check whether data are correctly using the function `head()` to check the first few (default is six) observations. (or)

```
mydata <- read.table("mydata.txt", header = TRUE, sep = "")
head(mydata)
# tail(mydata)
```

```
  Names Gender Weight Overweight
1   Bob      M   72.5      FALSE
2  John      M   83.1      FALSE
3  Anne      F   60.8      FALSE
4   Dan      M   89.7       TRUE
```

```
5   Juan      M   93.2       TRUE
6   Jane      F   76.9       TRUE
```

Often we may encounter data files ending with `.csv`, which is a special type of a text file, with commas separating each value. And we use the function `read.csv()` to read a `.csv` file.

```
tail(read.csv("mydata.csv", header = TRUE))
```

```
  Names Gender Weight Overweight
2  John      M   83.1      FALSE
3  Anne      F   60.8      FALSE
4   Dan      M   89.7       TRUE
5  Juan      M   93.2       TRUE
6  Jane      F   76.9       TRUE
7 Doris      F   56.3      FALSE
```

We can also read data from an excel file or a Stata file that we will see in the final section of this tutorial. To read these types of files, we need to load the packages of `gdata`, `foreign` (for Stata 12 and prior version), or `readstata13` (for Stata 13 and newer version).

```
library(gdata)
read.xls(mydata.xls)

library(foreign)
read.dta(mydata.dta)
```

3.3 Select variables

Since a data frame is a special case of list, we can select a variable in a data frame by using "`$`" or "`[[]]`". Here is an example of computing the average weight of students.

```
mean(mydata$Weight)
```

```
[1] 76.07143
```

3.4 Get summary information

After reading data into R, besides using `head()` or `tail()` to see the first and last few observations, we need also use `str()` and `summary()` to get some summary information of the data set.

```
str(mydata)
summary(mydata)
```

```
'data.frame': 7 obs. of  4 variables:
 $ Names     : Factor w/ 7 levels "Anne","Bob","Dan",..: 2 6 1 3 7 5 4
```

```
 $ Gender    : Factor w/ 2 levels "F","M": 2 2 1 2 2 1 1
 $ Weight    : num  72.5 83.1 60.8 89.7 93.2 76.9 56.3
 $ Overweight: logi  FALSE FALSE FALSE TRUE TRUE TRUE ...
   Names   Gender     Weight       Overweight
 Anne :1   F:3    Min.   :56.30   Mode :logical
 Bob  :1   M:4    1st Qu.:66.65   FALSE:4
 Dan  :1          Median :76.90   TRUE :3
 Doris:1          Mean   :76.07   NA's :0
 Jane :1          3rd Qu.:86.40
 John :1          Max.   :93.20
 Juan :1
```

The results of running `str()` show that the variables `Names` and `Gender` have the type of `Factor`. In default, when reading character variables from a file, R will convert them into factors that are categorical variables. We can preserve the type of character by including `stringsAsFactors=FALSE` in `read.table()` or `read.csv()`.

4 Graphics

R is very powerful in creating graphics. In this tutorial, we will learn base graphics systems in R.

We use a database, `mtcars`, in the `datasets` package in R to show how to draw different types of graphics. This data set contain the data that was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973–74 models).(Read `https://stat.ethz.ch/R-manual/R-devel/library/datasets/html/mtcars.html`)

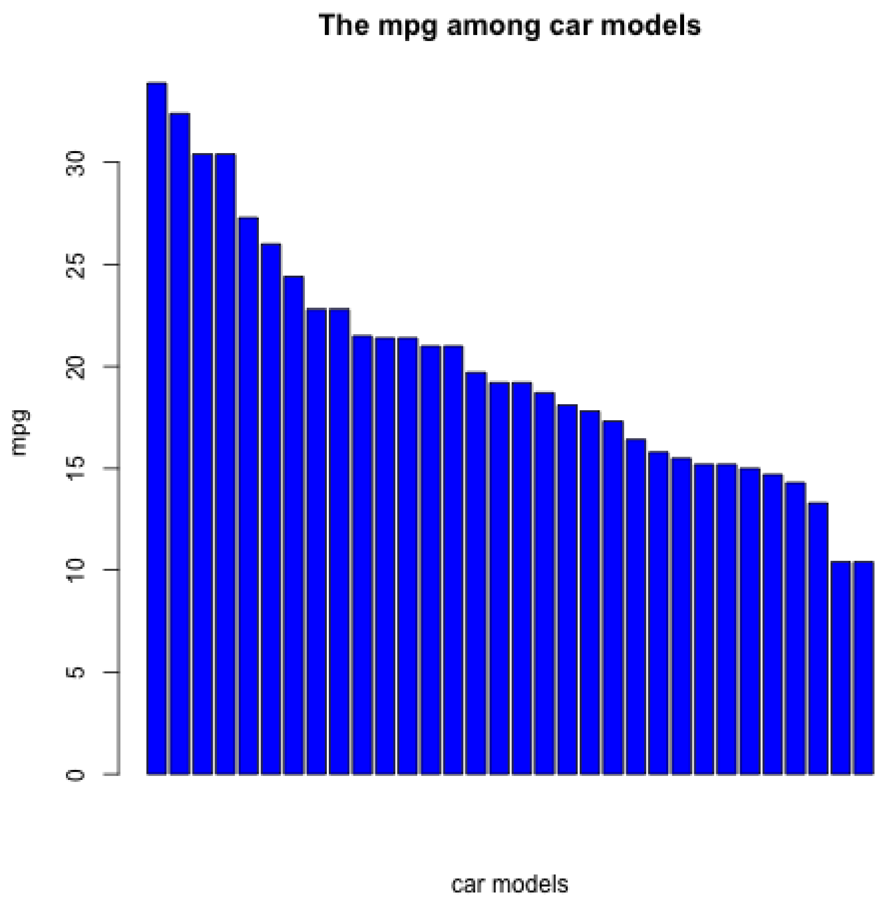
```
data(mtcars)
head(mtcars)
# str(mtcars)
```

```
                   mpg cyl disp  hp drat    wt  qsec vs am gear carb
Mazda RX4         21.0   6  160 110 3.90 2.620 16.46  0  1    4    4
Mazda RX4 Wag     21.0   6  160 110 3.90 2.875 17.02  0  1    4    4
Datsun 710        22.8   4  108  93 3.85 2.320 18.61  1  1    4    1
Hornet 4 Drive    21.4   6  258 110 3.08 3.215 19.44  1  0    3    1
Hornet Sportabout 18.7   8  360 175 3.15 3.440 17.02  0  0    3    2
Valiant           18.1   6  225 105 2.76 3.460 20.22  1  0    3    1
```

4.1 The barchart

First, Let's see the mpg (miles per gallon) among different models by the bar chart.

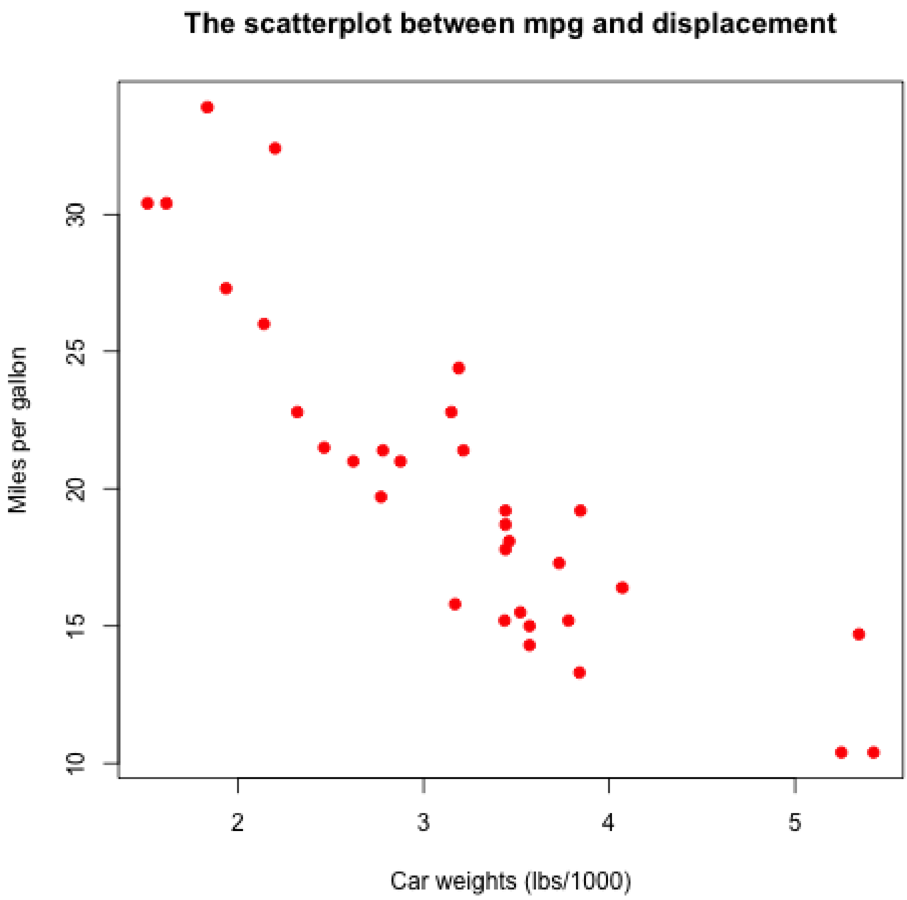
```
barplot(sort(mtcars$mpg, decreasing = TRUE),
col = "blue",
main = "The mpg among car models",
xlab = "car models", ylab = "mpg")
```



4.2 The scatterplot

We know in Lecture 3 that a scatterplot is often used to see the association between two variables. Let's see the relationship between miles per gallon, `mpg`, and car weights, `disp`.

```
plot(mtcars$wt, mtcars$mpg,
     main = "The scatterplot between mpg and displacement",
     xlab = "Car weights (lbs/1000)",
     ylab = "Miles per gallon",
     pch = 19, col = "red")
```



We will explore more graphic capabilities of R in the lectures to come.

5 Statistical Analysis

Now we can show how to use R to do some statistical analysis. This demonstration answers the questions of Empirical Exercise 3.1 at the end of Chapter 3. Furthermore, we carry out this exercise in the format of **reproducible research**. That means, we should accomplish they following tasks in answering the problem:

1. using R to compute the statistics asked in the questions
2. including R code and the results of running the code in the answer, and
3. describing our work and answers in plain language along with code and numerical answers.

5.1 A description of the problem

Empirical exercise 3.1 concerns the relationship between average earnings and education levels, using the data set from the 1992 and 2008 Current Population Survey (CPS). Specifically, we want to see whether the average hourly earnings (`ahe`) are different between workers with a bachelor degree and those with only high school diploma (`bachelor`).

5.2 Answers to the questions

Question (a)

> Compute the sample mean for average hourly earnings (`ahe`) in 1992 and in 2008. Construct a 95% confidence interval for the population means for `ahe` in 1992 and 2008 and the change between 1992 and 2008

- Read the data

  The first thing first is of course read the data correctly from the Stata file `data/cps92_08.dta`, which can be read by the function `read.dta()` in the package of `foreign`.

```
library(foreign)
cpsdat <- read.dta("data/cps92_08.dta")
head(cpsdat)
```

```
  year       ahe bachelor female age
1 1992 11.188811        1      0  29
2 1992 10.000000        1      0  33
3 1992  5.769231        0      0  30
4 1992  1.562500        0      0  32
5 1992 14.957265        1      0  31
6 1992  8.660096        1      1  26
```

- Calculate the sample means of average hourly earnings in 1992 and 2008

  There are many ways to compute the sample means in 1992 and 2008, respectively. First, to make you more familiar with the R language, we compute them in a very basic way. Then, we show how to get the same results with some powerful functions.

```
# extract the data for average hourly earnings in 1992 and 2008
ahe.92 <- cpsdat$ahe[cpsdat$year == 1992]
ahe.08 <- cpsdat$ahe[cpsdat$year == 2008]
mean.ahe.92 <- mean(ahe.92); mean.ahe.92
mean.ahe.08 <- mean(ahe.08); mean.ahe.08
```

```
[1] 11.62637
[1] 18.97609
```

  The average hourly earnings are `11.63` dollars in 1992 and `18.98` dollars in 2008.

- Construct the confidence intervals

Recall that a 95% confidence interval for the population mean can be constructed as $\overline{Y} \pm 1.96 SE(\overline{Y})$ and $SE(\overline{Y})$ is computed as $s_Y/\sqrt{n}$.

```
# the sample variance
sd.ahe.92 <- sd(ahe.92)
sd.ahe.08 <- sd(ahe.08)

n.92 <- length(ahe.92)
n.08 <- length(ahe.08)

# the standard error
se.ahe.92 <- sd.ahe.92 / sqrt(n.92)
se.ahe.08 <- sd.ahe.08 / sqrt(n.08)

# 95% confidence interval
# the 95% critical value from a normal distribution
cv.95 <- qnorm(0.975)

lower.lim.92 <- mean.ahe.92 - cv.95 * se.ahe.92
lower.lim.08 <- mean.ahe.08 - cv.95 * se.ahe.08

upper.lim.92 <- mean.ahe.92 + cv.95 * se.ahe.92
upper.lim.08 <- mean.ahe.08 + cv.95 * se.ahe.08
```

The 95% confidence interval for `ahe` in 1992 is (`11.5`, `11.75`), and that in 2008 is (`18.75`, `19.2`).

- Alternative methods to calculate the sample means and confidence intervals

  In the above example, to compute the sample averages in 1992 and 2008, we write code separately for each year, which can be done more easily in R.

  We can compute the averages for each year using the function `aggregate()`, which splits the whole data base into two parts by the values of `year`. Then, for each part we compute the average by specifying the argument `FUN` to be `mean`, i.e., specifying the function to be used for each part as the `mean()` function. Also, in this case, we use `~` to specify a **formula** that means that we split `ahe` by `year`.

```
# Use aggregate() to compute the means in both years
ahe.means <- aggregate(ahe ~ year, FUN = mean, data = cpsdat)
ahe.means
```

```
  year      ahe
1 1992 11.62637
2 2008 18.97609
```

The confidence interval can be extracted from the results of the `t.test()` function, which is a list.

```
# t test for ahe in 1992
t.ahe.92 <- t.test(ahe.92); t.ahe.92$conf.int
# t test for ahe in 2008
t.ahe.08 <- t.test(ahe.08); t.ahe.08$conf.int
# test for the change between 1992 and 2008
t.ahe.diff <- t.test(ahe.08, ahe.92); t.ahe.diff
```

```
[1] 11.50019 11.75254
attr(,"conf.level")
[1] 0.95
[1] 18.74975 19.20244
attr(,"conf.level")
[1] 0.95

Welch Two Sample t-test

data:  ahe.08 and ahe.92
t = 55.597, df = 12065, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 7.090601 7.608853
sample estimates:
mean of x mean of y
 18.97609  11.62637
```

The confidence interval of the change in average hourly earnings between 1992 and 2008 is (`7.09`, `7.61`).

Question (b)

Now we need to adjust the average hourly earnings in the 1992 dollars to the 2008 dollars with the inflation rate, computed as `CPI2008/CPI1992`.

```
# CPI in 1992 and 2008
cpi.92 <- 140.3
cpi.08 <- 215.2
# Inflation adjustment
inflator <- cpi.08 / cpi.92
cpsdat$ahe.adj <- with(cpsdat, ifelse(year == 1992, ahe * inflator, ahe))
```

In the code block above, we first use the function `with()` to attach the data frame `cpsdat` within its own environment so that when we refer to variables in `cpsdat`, such as `ahe` and `year`, we do not need to write `cpsdat$` and every time we use its variables.

The function `ifesle()` set the values of `ahe` based on the condition `year == 1992`. If the condition is true, we do `ahe * inflator`; if not, leave `ahe` as it is.

Then we repeat what we've done in Question (a) with the inflation-adjusted earnings in 1992.

```
ahe.92.adj <- with(cpsdat, ahe.adj[year == 1992])
mean.ahe.92.adj <- mean(ahe.92.adj)
t.ahe.92.adj <- t.test(ahe.92.adj)
t.ahe.diff.adj <- t.test(ahe.08, ahe.92.adj)
```

- The sample average of the inflation-adjusted earnings in 1992 is `17.83` in the 2008 dollars.
- The confidence interval for the inflation-adjusted average hourly earnings in 1992 is (`17.64, 18.03`).
- The confidence interval for the change between 1992 and 2008 is (`0.85, 1.44`).

Question (c)

If we are interested in the change in workers' purchasing power, the results with the inflation-adjusted earnings should be used in comparison.

Question (d)

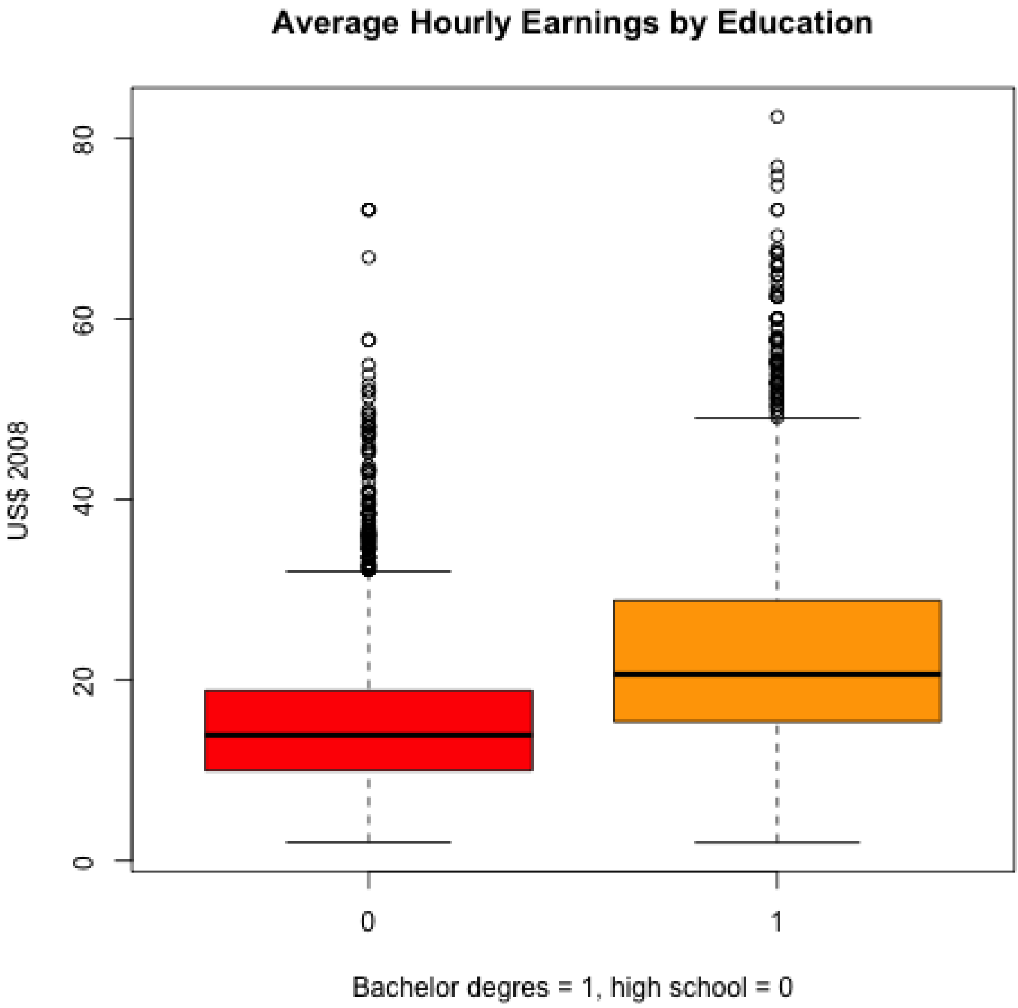
Now let's compute the average earnings for high school graduates and college graduates with the 2008 data. First thing to do is to select the 2008 data from `cpsdat` using the function `subset()`

```
# select data in 2008
cps08 <- subset(cpsdat, year == 2008, select = c(year, ahe, bachelor))

# calculate means
ahe.educ.08 <- aggregate(ahe ~ bachelor, FUN = mean, data = cps08)

# select ahe and filter by bachelor
ahe.high.08 <- with(cps08, ahe[bachelor == 0])
ahe.bach.08 <- with(cps08, ahe[bachelor == 1])

# construct confindence interval
t.ahe.high.08 <- t.test(ahe.high.08)
t.ahe.bach.08 <- t.test(ahe.bach.08)
t.ahe.gap.08  <- t.test(ahe.bach.08, ahe.high.08)
```

- The mean of the average hourly earnings of high school graduates in 2008 is `15.33` dollars with the 95% confidence interval (`15.09, 15.57`)
- The mean of the average hourly earnings of college graduates is `22.91` dollars with the 95% confidence interval (`22.56, 23.26`)
- The 95% confidence interval of the gap in earnings between the two groups is (`7.15, 8`)

We can create a boxplot to compare the means and confidence intervals of average hourly earnings between high school graduates and college graduates.

```
boxplot(ahe ~ bachelor, data = cps08,
main = "Average Hourly Earnings by Education",
col = c("red", "orange"),
xlab = "Bachelor degres = 1, high school = 0",
ylab = "US$ 2008")
```



We leave Question (e)-(g) to students as exercises.

I include all the files to generate a complete answer to Empirical exercise 3.1 in the following package, `rfiles.zip`, including the R code file, Rmarkdown file, the data file, and the html and pdf files containing the answers.